The Higgs sector of a supersymmetric left-right model

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Abstract

We study the symmetry breaking sector of a supersymmetric left-right model based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The explicit mass matrices of neutral, singly charged and doubly charged scalars are constructed. In the minimum the R-parity is found to be spontaneously broken. An experimentally interesting feature of the model is that one of the doubly charged scalars is possibly light enough to be seen in the next linear collider.

1. Introduction. Supersymmetry is often invoked to take care of the quadratic divergences occurring in the scalar sector of spontaneously broken gauge theories. The Higgs sector of supersymmetric models has proven to be interesting in view of future colliders, since the mass of the lightest neutral scalar has typically a relatively low upper limit. In the framework of the minimal supersymmetric standard model (MSSM) the tree level mass of the neutral Higgs is bound from above by the Z boson mass. However, radiative corrections to the scalar masses can be large [1].

In this paper, we will study the Higgs sector of a supersymmetric left-right model (SLRM) based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. We construct the mass matrices of the physical Higgs scalars and determine the mass spectra for a representative choice of free parameters.

The motivation for the left-right models is mainly the see-saw mechanism [2] by which one can generate light masses for the left-handed neutrinos and large masses for the right-handed ones. The left-right models are especially interesting, if the experiments on solar [3] and atmospheric [4] neutrinos continue to show deviation from the standard model, as well as the existence of the hot dark matter component [5] explaining some features of the power spectrum of density fluctuations of the Universe persists. All these results seem to indicate that neutrinos indeed have a small mass.

To achieve the see-saw mechanism, the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry has to be broken by scalar triplets of $SU(2)_R$. The special feature of the model is that the triplets contain among others also doubly charged scalars. The phenomenology related to the supersymmetric partners of these scalars has been recently studied in [6]. The gauge symmetry breaking of the supersymmetric left-right model was also studied in [7, 8]. In [8] it was argued that the parity is violated only if the R-parity is broken. Here we will see in an explicit construction that the tree-level masses of the pseudoscalars and doubly charged scalars can be physical only if the R-parity is spontaneously broken.

2. The scalar potential of a supersymmetric left-right model and the gauge symmetry breaking. The most general potential [9] of the standard left-right model is complicated due to the numerous possible combinations of the fields. In the supersymmetric version, the Higgs couplings are much more constrained, since the quartic interactions are completely determined by the gauge couplings.

The superpotential of the model is given by

$$W = h_{\phi Q} \widehat{Q}_{L}^{T} i \tau_{2} \widehat{\phi} \widehat{Q}_{R}^{c} + h_{\chi Q} \widehat{Q}_{L}^{T} i \tau_{2} \widehat{\chi} \widehat{Q}_{R}^{c}$$

$$+ h_{\phi L} \widehat{L}_{L}^{T} i \tau_{2} \widehat{\phi} \widehat{L}_{R}^{c} + h_{\chi L} \widehat{L}_{L}^{T} i \tau_{2} \widehat{\chi} \widehat{L}_{R}^{c} + h_{\Delta} \widehat{L}_{R}^{cT} i \tau_{2} \widehat{\Delta} \widehat{L}_{R}^{c}$$

$$+ \mu_{1} \operatorname{Tr} (i \tau_{2} \widehat{\phi}^{T} i \tau_{2} \widehat{\chi}) + \mu_{2} \operatorname{Tr} (\widehat{\Delta} \widehat{\delta}),$$

$$(1)$$

where $\hat{Q}_{L(R)}$ denote the left (right) handed quark superfield doublets and similarly for the leptons $\hat{L}_{L(R)}$. The triplet and the bidoublet Higgs superfields of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ are given by

$$\widehat{\Delta} = \begin{pmatrix} \widehat{\Delta}^{-}/\sqrt{2} & \widehat{\Delta}^{0} \\ \widehat{\Delta}^{--} & -\widehat{\Delta}^{-}/\sqrt{2} \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -2),$$

$$\widehat{\delta} = \begin{pmatrix} \widehat{\delta}^{+}/\sqrt{2} & \widehat{\delta}^{++} \\ \widehat{\delta}^{0} & -\widehat{\delta}^{+}/\sqrt{2} \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, 2),$$

$$\widehat{\phi} = \begin{pmatrix} \widehat{\phi}_{1}^{0} & \widehat{\phi}_{1}^{+} \\ \widehat{\phi}_{2}^{-} & \widehat{\phi}_{2}^{0} \end{pmatrix} \sim (\mathbf{2}, \mathbf{2}, 0), \qquad \widehat{\chi} = \begin{pmatrix} \widehat{\chi}_{1}^{0} & \widehat{\chi}_{1}^{+} \\ \widehat{\chi}_{2}^{-} & \widehat{\chi}_{2}^{0} \end{pmatrix} \sim (\mathbf{2}, \mathbf{2}, 0). \tag{2}$$

Corresponding to each scalar multiplet with non-zero U(1) quantum number, one has to include another multiplet with an opposite U(1) quantum number in order to avoid chiral anomalies for the fermionic superpartners. Also another bidoublet Higgs superfield is added to get a nontrivial Kobayashi-Maskawa matrix.

The left-right model contains often also the $SU(2)_L$ triplet so as to make the Lagrangian fully symmetric under the $L \leftrightarrow R$ transformation. Phenomenologically this is not, however, necessary, since the left triplet is not needed for the symmetry breaking or the see-saw mechanism. As our purpose is to study a minimal phenomenologically viable model, we have not included the $SU(2)_L$ triplets in our considerations.

To explore the symmetry breaking and to work out the mass spectra of the Higgs sector, one has to consider the so-called F-terms and D-terms, as well as the possible soft supersymmetry breaking terms. Those of such terms which contain electrically neutral scalars are given by

$$V_{D} = \sum_{i} \left\{ \frac{1}{2} g_{L}^{2} \left| \frac{1}{2} \operatorname{Tr} \phi^{\dagger} \tau_{i} \phi + \frac{1}{2} \operatorname{Tr} \chi^{\dagger} \tau_{i} \chi + \frac{1}{2} \tilde{L}_{L}^{\dagger} \tau_{i} \tilde{L}_{L} \right|^{2} \right.$$

$$\left. + \frac{1}{2} g_{R}^{2} \left| \frac{1}{2} \operatorname{Tr} \phi^{\dagger} \tau_{i} \phi + \frac{1}{2} \operatorname{Tr} \chi^{\dagger} \tau_{i} \chi + \operatorname{Tr} \Delta^{\dagger} \tau_{i} \Delta + \operatorname{Tr} \delta^{\dagger} \tau_{i} \delta + \frac{1}{2} \tilde{L}_{R}^{*\dagger} \tau_{i} \tilde{L}_{R}^{*} \right|^{2} \right\}$$

$$\left. + \frac{1}{2} g_{B-L}^{2} \left| - \operatorname{Tr} \Delta^{\dagger} \Delta + \operatorname{Tr} \delta^{\dagger} \delta - \frac{1}{2} \tilde{L}_{L}^{\dagger} \tilde{L}_{L} + \frac{1}{2} \tilde{L}_{R}^{*\dagger} \tilde{L}_{R}^{*} \right|^{2},$$

$$V_{soft} + V_{F} =$$

$$m_{1}^{2} \operatorname{Tr} |\phi|^{2} + m_{2}^{2} \operatorname{Tr} |\chi|^{2} - (m_{\phi \chi}^{2} \operatorname{Tr} (i \tau_{2} \phi^{T} i \tau_{2}) \chi + h.c.)$$

$$+ m_{2}^{2} |\Delta|^{2} + m_{4}^{2} |\delta|^{2} - (m_{\phi \chi}^{2} \operatorname{Tr} \Delta \delta + h.c.) + m_{5}^{2} |\tilde{L}_{R}^{*}|^{2} + m_{6}^{2} |\tilde{L}_{L}|^{2}$$

$$\begin{split} & m_{1}^{2} \mathrm{Tr} |\phi|^{2} + m_{2}^{2} \mathrm{Tr} |\chi|^{2} - (m_{\phi\chi}^{2} \mathrm{Tr} (i\tau_{2}\phi^{T} i\tau_{2})\chi + h.c.) \\ & + m_{3}^{2} |\Delta|^{2} + m_{4}^{2} |\delta|^{2} - (m_{\Delta\delta}^{2} \mathrm{Tr} \Delta \delta + h.c.) + m_{5}^{2} |\tilde{L}_{R}^{*}|^{2} + m_{6}^{2} |\tilde{L}_{L}|^{2} \\ & + (\tilde{L}_{R}^{T} (A_{\phi}\phi + A_{\chi}\chi)\tilde{L}_{L} + A_{\Delta}\tilde{L}_{R}^{*T} i\tau_{2}\Delta\tilde{L}_{R}^{*} + h.c.) \\ & + |h_{\phi L} i\tau_{2}\phi\tilde{L}_{R}^{*} + h_{\chi L} i\tau_{2}\chi\tilde{L}_{R}^{*}|^{2} + |h_{\phi L}\tilde{L}_{L}^{T} i\tau_{2}\phi + h_{\chi L}\tilde{L}_{L}^{T} i\tau_{2}\chi + 2h_{\Delta}\tilde{L}_{R}^{*T} i\tau_{2}\Delta|^{2} \\ & + \mathrm{Tr}|h_{\Delta}\tilde{L}_{R}^{*} (\tilde{L}_{R}^{*T} i\tau_{2}) + \mu_{2}\delta|^{2} + |-h_{\chi L} i\tau_{2}\tilde{L}_{L}\tilde{L}_{R}^{*T} + \mu_{1} i\tau_{2}\phi i\tau_{2}|^{2} \\ & + |-h_{\phi L} i\tau_{2}\tilde{L}_{L}\tilde{L}_{R}^{*T} + \mu_{1} i\tau_{2}\chi i\tau_{2}|^{2}, \end{split} \tag{3}$$

where the scalar fields are denoted by the same symbols than the superfields, except for the hat. The soft supersymmetry breaking parameters are the soft trilinear couplings A_i and the soft masses which are contained in m_i . The m_i 's are defined so that the coefficients of the F-terms of similar form are included.

We do not make an effort to minimize the full scalar potential, but instead find a region in the parameter space for which the scalar fields in the minimum have vacuum expectation values given as follows:

$$\langle \phi \rangle = \begin{pmatrix} \kappa_1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \langle \chi \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \kappa_2 \end{pmatrix},$$

$$\langle \Delta \rangle = \begin{pmatrix} 0 & v_{\Delta} \\ 0 & 0 \end{pmatrix}, \qquad \langle \delta \rangle = \begin{pmatrix} 0 & 0 \\ v_{\delta} & 0 \end{pmatrix}$$
$$\langle \tilde{\nu}_{R} \rangle = \begin{pmatrix} 0 \\ \sigma_{R} \end{pmatrix}, \qquad \langle \tilde{\nu}_{L} \rangle = \begin{pmatrix} \sigma_{L} \\ 0 \end{pmatrix}. \tag{4}$$

Also ϕ_2^0 and χ_1^0 in Eq. (2) could get vev's without breaking the electric charge, but as explained in [6] on phenomenological grounds $\langle \phi_2^0 \rangle$, $\langle \chi_1^0 \rangle$ are much smaller than $\langle \phi_1^0 \rangle$, $\langle \chi_2^0 \rangle$, since the mixing between the charged gauge bosons has to be tiny.

The Yukawa coupling $h_{\chi L}$ is proportional to the neutrino Dirac mass m_D . The light neutrino mass in the see-saw mechanism is given by $\sim m_D^2/m_M$, where $m_M = h_{\Delta}v_{\Delta}$ is the Majorana mass. The value of the triplet Higgs vev v_{Δ} gives the scale of the SU(2)_R breaking and is, according to the lower bounds of the heavy W- and Z-boson masses, in the range $v_{\Delta} \gtrsim 1$ TeV (see below). Therefore the magnitude of $h_{\chi L}$ is still quite inaccurately determined given the present upper limits for the light neutrino masses. On the other hand the Yukawa coupling $h_{\phi L}$ is proportional to the electron mass and we will neglect it.

The m_i 's, i=1,...,5 appearing in (3), can be eliminated by using the minimization conditions $\partial V/\partial \kappa_2 = \partial V/\partial \kappa_1 = \partial V/\partial v_\Delta = \partial V/\partial v_\delta = \partial V/\partial \sigma_R = \partial V/\partial \sigma_L = 0$. For simplicity we will assume in the following that only one of the vev's of the neutral scalar leptons is non-negligible, namely σ_R . This will fix the parameter μ_1 as $\mu_1 = 2h_\Delta \kappa_2 v_\Delta/\kappa_1$. Imposing the conditions for minimum, the potential is given as

$$V|_{min} = -\frac{1}{8}g_L^2(\kappa_1^2 - \kappa_2^2)^2 - \frac{1}{2}g_{B-L}^2(v_\Delta^2 - v_\delta^2 - \frac{1}{2}\sigma_R^2)^2 - \frac{1}{2}g_R^2(v_\Delta^2 - v_\delta^2 - \frac{1}{2}\sigma_R^2 - \frac{1}{2}(\kappa_1^2 - \kappa_2^2))^2 + \sigma_R^2(-4h_\Delta^2v_\Delta^2 - h_\Delta^2\sigma_R^2 - h_{\chi L}^2\kappa_2^2 + A_\Delta v_\Delta + h_\Delta \mu_2 v_\delta).$$
(5)

In the true minimum this has to be negative. For the first three terms this is obvious, but the last term is constrained by the requirement of negativity.

In the breakdown of the gauge symmetries down to the $U(1)_{em}$ the charged gauge bosons and two of the neutral ones become massive. The masses are found by diagonalizing the corresponding mass matrices and they are given by

$$m_{Z_1}^2 = \frac{1}{2} (g_L^2 + g'^2) (\kappa_1^2 + \kappa_2^2) + O\left(\frac{(\kappa_1^2 + \kappa_2^2)^2}{v_\Lambda^2 + v_\delta^2 + \frac{1}{4}\sigma_R^2}\right), \tag{6}$$

$$m_{W_L}^2 = \frac{1}{2}g_L^2(\kappa_1^2 + \kappa_2^2), \tag{7}$$

$$m_{Z_2}^2 = 2g_R^2 \left[\frac{1}{4} \frac{g'^2}{g_{B-L}^2} (\kappa_1^2 + \kappa_2^2) + \frac{g_{B-L}^2}{g'^2} (v_\Delta^2 + v_\delta^2 + \frac{1}{4} \sigma_R^2) \right] + O\left(\frac{(\kappa_1^2 + \kappa_2^2)^2}{v_\Delta^2 + v_\delta^2 + \frac{1}{4} \sigma_R^2} \right), \tag{8}$$

$$m_{W_R}^2 = g_R^2 (v_\Delta^2 + v_\delta^2 + \frac{1}{2} \sigma_R^2 + \frac{1}{2} (\kappa_1^2 + \kappa_2^2)). \tag{9}$$

Here it is denoted $g' = g_R g_{B-L} / \sqrt{g_R^2 + g_{B-L}^2}$

In the following we will assume that the left- and right-couplings are equal to the standard model SU(2) gauge coupling g, $g_L = g_R = g$. This assumption also determines the value of the third coupling, g' by (6) up to a correction factor. Then the ρ -parameter of the electroweak interactions is found to be given by

$$\rho^{-1} = \frac{M_{Z_1}^2 \cos^2 \theta_W}{M_{W_L}^2}$$

$$= 1 - \frac{1}{4} \frac{g'^4}{g_{B-L}^4} \frac{\kappa_1^2 + \kappa_2^2}{v_\Delta^2 + v_\delta^2 + \frac{1}{4} \sigma_R^2} + O\left(\frac{(\kappa_1^2 + \kappa_2^2)^2}{(v_\Delta^2 + v_\delta^2 + \frac{1}{4} \sigma_R^2)^2}\right), \tag{10}$$

where the angle θ_W is given by $\tan \theta_W = g'/g$. The effect of the new scalars getting vev's is to increase the value of ρ . As expected, the value of the ρ parameter approaches its standard model value as v_{Δ} , v_{δ} , or σ_R get large values. The experimental limits for the ρ -parameter are given by $\rho = 0.998 \pm 0.0086$ [10]. With the above mentioned assumptions on the gauge couplings, the experimental bound for ρ constrains the ratio of the vacuum expectation values as follows

$$\frac{\kappa_1^2 + \kappa_2^2}{v_\Delta^2 + v_\delta^2 + \frac{1}{4}\sigma_R^2} < 0.053. \tag{11}$$

Since the sum of the vacuum expectation values $\kappa_{1,2}^2$ is determined from the measured mass of W_L [11], the charged heavy gauge boson mass $m_{W_R} \gtrsim 500$ GeV.

3. Spontaneous breaking of R-parity. In the minimum the masses of all the scalars in the Higgs sector must be positive. The scalar masses can be found by using the scalar potential, Eq. (3), and the vacuum expectation values given in Eq. (4). Before studying the ensuing mass spectrum numerically, let us consider the R-parity, $R = (-1)^{3(B-L)+2s}$. As discussed in [12], the R-parity, which is +1 for ordinary particles and -1 for their supersymmetric counterparts, is automatically conserved in Lagrangian in this type of models, but it may be broken spontaneously if $\langle \tilde{\nu} \rangle \neq 0$. In the neutral mass matrix the pseudoscalar and scalar components do not mix. In the case of conserved R-parity, i.e. $\langle \tilde{\nu}_{R,L} \rangle = 0$, the pseudoscalar mass matrix is given by four two by two blocks, see Eqs. (A5) and (A6) in the Appendix. One of the blocks contains the sneutrinos and we need not consider it here. Two of the blocks contain the Goldstone bosons which make two of the neutral gauge bosons massive. The physical pseudoscalar particles have the masses

$$m_{A_{1}}^{2} = m_{\phi\chi}^{2} \left(\frac{\kappa_{1}}{\kappa_{2}} + \frac{\kappa_{2}}{\kappa_{1}}\right),$$

$$m_{A_{2}}^{2} = m_{\Delta\delta}^{2} \left(\frac{v_{\delta}}{v_{\Delta}} + \frac{v_{\Delta}}{v_{\delta}}\right),$$

$$m_{A_{3,4}} = \frac{1}{2} \left\{ m_{A_{1}}^{2} \pm \left[m_{A_{1}}^{4} + 4(m_{W_{R}}^{2} \cos 2\gamma - m_{W_{L}}^{2} \cos 2\beta)^{2} - 4(m_{W_{R}}^{2} \cos 2\gamma - m_{W_{L}}^{2} \cos 2\beta) m_{A_{1}}^{2} \cos 2\beta \right]^{1/2} \right\},$$

$$(12)$$

where we have defined

$$\tan \beta = \frac{\kappa_2}{\kappa_1}, \quad \tan \delta = \frac{v_\delta}{v_\Delta}, \text{ and } \tan^2 \gamma = \frac{v_\delta^2 + \frac{1}{2}\kappa_1^2}{v_\Delta^2 + \frac{1}{2}\kappa_2^2}.$$
 (13)

On the other hand the masses of the doubly charged scalars are given by

$$m_{H_{1,2}^{++}}^2 = \frac{1}{2} \left\{ m_{A_2}^2 \pm \sqrt{m_{A_2}^4 + 8m_{W_R}^2 \cos 2\gamma [m_{A_2}^2 \cos 2\delta + 2m_{W_R}^2 \cos 2\gamma]} \right\}.$$
 (14)

It is easily seen that trying to make both pseudoscalar and doubly charged masses positive one ends in contradiction. Necessarily at least one of the $\langle \tilde{\nu} \rangle \neq 0$.

4. Scalar mass spectrum. Let us now investigate the mass spectrum of the scalars predicted by the model. The physical Higgses in the model consist of eight neutral scalars, six pseudoscalars, six singly charged scalars, and two doubly charged scalars. The corresponding mass matrices are given in the Appendix in Eqs. (A3) - (A9). Throughout the numerical calculations it is assumed that $g_L = g_R = g$ and the mass of W_R is taken to be 1 TeV. In this case the value of the ρ -parameter increases from its standard model value by $\Delta \rho = \rho - 1 \sim 0.0017$.

The mass parameters containing the soft breaking terms are chosen to be 1 TeV. As usual in the susy models, it turns out that the experimentally most interesting, lightest scalar masses, both neutral and singly or doubly charged, depend only very slightly on the soft masses. E.g. changing $m_{\phi\chi}$ and $m_{\Delta\delta}$ from 1 TeV to 2 TeV increases the light masses by less than 5 GeV. We have also assumed that $m_5 = m_6$.

The parameters of the model are constrained by the requirement that all the masses remain real in the allowed range. Reality of the pseudoscalar masses leads to an upper limit of $h_{\chi L}$ which varies between about 0.2 and 0.4. To obtain more insight into the parameters of the mass matrices, we study first a specific limit with $h_{\chi L} = 0 = \mu_2$ and D-terms, which are negligible. In this limit the doubly charged masses are real if $A_{\Delta} > 4h_{\Delta}^2 v_{\Delta}$. On the other hand one finds that the neutral scalar masses can be real only if A_{Δ} is in the range

$$\frac{h_{\Delta}^{2}}{2v_{\Delta}} \left[\sigma_{R}^{2} + 8v_{\Delta}^{2} - \sqrt{\sigma_{R}^{4} + 16\sigma_{R}^{2}v_{\Delta}^{2}} \right] < A_{\Delta} < \frac{h_{\Delta}^{2}}{2v_{\Delta}} \left[\sigma_{R}^{2} + 8v_{\Delta}^{2} + \sqrt{\sigma_{R}^{4} + 16\sigma_{R}^{2}v_{\Delta}^{2}} \right].$$
(15)

With these restrictions the masses of pseudoscalars and singly charged scalars are real. The allowed range disappears in the limit $h_{\Delta} \to 0$. In a more general situation with $h_{\chi L} = 0.1$ and g = 0.65 the coupling A_{Δ} as a function of h_{Δ} is plotted in Fig. 1 for two values of the supersymmetric mass parameter μ_2 ($\mu_2 = 100$ GeV and 1 TeV). The masses of the doubly charged scalars are positive above the lower curve, whereas the masses of the neutral scalars are positive below the upper curve. The relatively narrow range between the curves is thus the allowed range of h_{Δ} and A_{Δ} . For very small values of h_{Δ} this range almost disappears. Here we have taken $\tan \beta = 50$, $v_{\delta}/\sigma_R = 1.5$, and $v_{\Delta}/v_{\delta} = 1.05$. The curves depend only very slightly

on $\tan \beta$.

In Table 1 and 2 we show a typical spectra of the physical scalars and give their compositions for $\tan \beta = 1.5$ and $\tan \beta = 50$. We also give compositions of the unphysical Goldstone bosons needed to get massive gauge bosons. The lightest neutral scalar contains mostly bidoublet fields ϕ_1^{0r} and χ_2^{0r} whereas the heaviest is mostly a combination of the triplet fields Δ^{0r} , δ^{0r} , and the right-sneutrino $\tilde{\nu}_R^r$. Similar compositions are found for the lightest and heaviest pseudoscalars and the singly charged scalars. Varying the supersymmetric mass parameter μ_2 between 100 GeV and 1 TeV has an effect of a few GeV on the mass of the lightest neutral Higgs, whereas the effect on the lightest singly charged Higgs is negligible. The lightest neutral scalar resembles closely the lightest neutral scalar of MSSM. Consequently, the radiative corrections to $m_{H_1^0}$ are large due to the top and stop loops [1]. One may expect also large radiative corrections to the masses of those neutral scalars, which contain large portions of Δ^0 , due to the heavy right-handed neutrino contributions. However, the scalars containing Δ^0 tend to be heavy already in the tree level.

From the experimental point of view the most interesting situation arises when the doubly charged scalar is light. It is easy to detect (the signature is two leptons of same charge) and it is typical for the left-right model. The variation of the H^{++} masses as a function of the allowed A_{Δ} values is plotted in Fig. 2 for $h_{\Delta} = 0.3, \ldots, 0.8$. The mass increases fast from zero as A_{Δ} increases. The solid curve corresponds again to $\mu_2 = 1$ TeV and the dashed one to $\mu_2 = 100$ GeV. Increasing μ_2 changes the place of the curve in A_{Δ} axis, but the behaviour of the mass with increasing A_{Δ} is very similar. The maximum value of $m_{H^{++}}$ is typically between 200 GeV and 400 GeV.

The relation between the mass of the doubly charged Higgs and the possibly non-diagonal Yukawa coupling to the leptons have been studied in [13, 14]. The most stringent constraint comes from the upper limit for the decay $\mu \to \overline{e}ee$ [13]:

$$h_{\Delta,e\mu}h_{\Delta,ee} < 4.7 \times 10^{-11} \text{GeV}^{-2} \times m_{H^{++}}^2.$$
 (16)

From the Bhabha-scattering cross section at SLAC and DESY the following bound for the $h_{\Delta,ee}$ coupling was established:

$$h_{\Delta,ee}^2 < 9.7 \times 10^{-6} \text{GeV}^{-2} \times m_{H^{++}}^2.$$
 (17)

For $h_{\Delta,ee}$ =0.6 the mass of the doubly charged boson $m_{H^{++}} \gtrsim 200$ GeV. For the coupling $h_{\Delta,\mu\mu}$ the muonium transformation to antimuonium converts into a limit $h_{\Delta,ee}h_{\Delta,\mu\mu} < 5.8 \times 10^{-5} \text{GeV}^{-2} \times m_{H^{++}}^2$ [13].

5. Summary. The physical Higgses in the supersymmetric $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ electroweak model we have studied consist of eight neutral scalars, six pseudoscalars, six singly charged scalars, and two doubly charged scalars. We have shown that non-vanishing vacuum expectation values of bidoublet and triplet Higgses, sufficent for the breaking of the gauge symmetry, do not produce a physical mass spectrum, but also at least one of the sneutrinos should acquire a vev. Hence R-parity is necessarily broken in the model. With a viable choice of the various mass parameters we find that there are a few relatively light scalars or pseudoscalars with masses of the order of 100-200 GeV. These include a phenomenologically quite interesting doubly charged scalar with a mass about 200 GeV. The majority of the scalar particles are in the mass range above 1 TeV.

Appendix.

In this Appendix we give the mass matrices of neutral, singly charged and doubly charged scalars. In the mass matrices we denote

$$\kappa_1^2 - \kappa_2^2 = \kappa_{dif}^2,\tag{A1}$$

$$v_{\Delta}^2 - v_{\delta}^2 - \frac{1}{2}\sigma_R^2 - \frac{1}{2}(\kappa_1^2 - \kappa_2^2) = \rho_{dif}^2.$$
 (A2)

In the neutral mass matrix the pseudoscalar and scalar components do not mix. We denote the pseudoscalars by a superscript i and scalars by r. The mass matrix of the neutral scalar sector consists of one 2×2 and one 6×6 block:

$$M_{\phi_{2}^{0r},\chi_{1}^{0r}}^{2} = \begin{pmatrix} m_{\phi\chi}^{2} \frac{\kappa_{2}}{\kappa_{1}} - \frac{1}{2}g_{L}^{2}\kappa_{dif}^{2} + g_{R}^{2}\rho_{dif}^{2} & -m_{\phi\chi}^{2} \\ -m_{\phi\chi}^{2} & m_{\phi\chi}^{2} \frac{\kappa_{1}}{\kappa_{2}} + \frac{1}{2}g_{L}^{2}\kappa_{dif}^{2} - g_{R}^{2}\rho_{dif}^{2} - h_{\chi L}^{2}\sigma_{R}^{2} \end{pmatrix}, \tag{A3}$$

and $(M^2_{\tilde{\nu}_{I}^{r}, \tilde{\nu}_{P}^{*r}, \phi_{1}^{0r}, \chi_{2}^{0r}, \Delta^{0r}, \delta^{0r}})_{ij} = (M^{0,r(6)})_{ij}$ where the non-zero terms are given by

$$(M^{0,r(6)})_{\tilde{\nu}_L\tilde{\nu}_L} = m_6^2 + h_{\chi L}^2(\kappa_2^2 + \sigma_R^2) + \frac{1}{4}(g_L^2 + g_{B-L}^2)\kappa_{dif}^2 + \frac{1}{2}g_{B-L}^2\rho_{dif}^2,$$

$$(M^{0,r(6)})_{\tilde{\nu}_L\tilde{\nu}_R^*} = -2h_{\chi L}h_{\Delta}\kappa_2v_{\Delta} + h_{\chi L}\mu_1\kappa_1,$$

$$(M^{0,r(6)})_{\tilde{\nu}_L\phi_2^0} = h_{\chi L}\mu_1\sigma_R,$$

$$(M^{0,r(6)})_{\tilde{\nu}_L\chi_2^0} = -2h_{\chi L}h_{\Delta}\sigma_Rv_{\Delta},$$

$$(M^{0,r(6)})_{\tilde{\nu}_L\lambda^0} = -2h_{\chi L}h_{\Delta}\sigma_R\kappa_2,$$

$$(M^{0,r(6)})_{\tilde{\nu}_R^*\tilde{\nu}_R^*} = 4h_{\Delta}^2\sigma_R^2 + \frac{1}{2}(g_{B-L}^2 + g_R^2)\sigma_R^2,$$

$$(M^{0,r(6)})_{\tilde{\nu}_R^*\phi_1^0} = \frac{1}{2}g_R^2\kappa_1\sigma_R,$$

$$(M^{0,r(6)})_{\tilde{\nu}_R^*\phi_1^0} = (2h_{\chi L}^2 - \frac{1}{2}g_R^2)\sigma_R\kappa_2,$$

$$(M^{0,r(6)})_{\tilde{\nu}_R^*\phi_1^0} = (g_R^2 + g_{B-L}^2)v_{\delta}\sigma_R - 2h_{\Delta}\mu_2\sigma_R,$$

$$(M^{0,r(6)})_{\tilde{\nu}_R^*\delta^0} = (g_R^2 + g_{B-L}^2)v_{\delta}\sigma_R - 2h_{\Delta}\mu_2\sigma_R,$$

$$(M^{0,r(6)})_{\phi_1^0\phi_1^0} = m_{\phi\chi}^2\frac{\kappa_2}{\kappa_1} + \frac{1}{2}(g_R^2 + g_L^2)\kappa_1^2,$$

$$(M^{0,r(6)})_{\phi_1^0\lambda_2^0} = -m_{\phi\chi}^2 - \frac{1}{2}(g_R^2 + g_L^2)\kappa_1\kappa_2,$$

$$(M^{0,r(6)})_{\phi_1^0\delta^0} = g_R^2\kappa_1v_{\delta},$$

$$(M^{0,r(6)})_{\phi_1^0\delta^0} = g_R^2\kappa_1v_{\delta},$$

$$(M^{0,r(6)})_{\phi_2^0\delta^0} = g_R^2\kappa_2v_{\delta},$$

$$(M^{0,r(6)})_{\chi_2^0\sigma^0} = g_R^2\kappa_2v_{\delta},$$

$$(M^{0,r(6)})_{\lambda_0^0\delta^0} = -g_R^2\kappa_2v_{\delta},$$

$$(M^{0,r(6)})_{\Delta^0\delta^0} = m_{\Delta}^2\frac{v_{\delta}}{v_{\Delta}} + A_{\Delta}\frac{\sigma_R^2}{v_{\delta}} + 2(g_R^2 + g_{B-L}^2)v_{\delta}^2.$$

$$(M^{0,r(6)})_{\delta^0\delta^0} = m_{\Delta}^2\frac{v_{\delta}}{v_{\Delta}} + h_{\Delta}\mu_2\frac{\sigma_R^2}{v_{\omega}} + 2(g_R^2 + g_{B-L}^2)v_{\delta}^2.$$

$$(A^0)^{r(6)})_{\delta^0\delta^0} = m_{\Delta}^2\frac{v_{\delta}}{v_{\Delta}} + h_{\Delta}\mu_2\frac{\sigma_R^2}{v_{\omega}} + 2(g_R^2 + g_{B-L}^2)v_{\delta}^2.$$

The pseudoscalar mass matrix is of block diagonal form of one two by two block and one six by six block:

$$M_{\phi_2^{0i},\chi_1^{0i}}^2 = \begin{pmatrix} m_{\phi\chi}^2 \frac{\kappa_2}{\kappa_1} - \frac{1}{2} g_L^2 \kappa_{dif}^2 + g_R^2 \rho_{dif}^2 & m_{\phi\chi}^2 \\ m_{\phi\chi}^2 & m_{\phi\chi}^2 \frac{\kappa_1}{\kappa_2} + \frac{1}{2} g_L^2 \kappa_{dif}^2 - g_R^2 \rho_{dif}^2 - h_{\chi L}^2 \sigma_R^2 \end{pmatrix}$$
(A5)

and $(M^2_{\tilde{\nu}_L^i, \tilde{\nu}_R^{*i}, \phi_1^{0i}, \chi_2^{0i}, \Delta^{0i}, \delta^{0i}})_{ij} = (M^{0,i(6)})_{ij}$, with the non-zero terms given by

$$(M^{0,i(6)})_{\tilde{\nu}_L\tilde{\nu}_L} = m_6^2 + h_{\chi_L}^2(\kappa_2 + \sigma_R^2) + \frac{1}{4}(g_L^2 + g_{B-L}^2)\kappa_{dif}^2 + \frac{1}{2}g_{B-L}^2\rho_{dif}^2,$$

$$(M^{0,i(6)})_{\tilde{\nu}_L\tilde{\nu}_R^*} = -2h_{\chi_L}h_{\Delta}\kappa_2v_{\Delta} - h_{\chi_L}\mu_1\kappa_1,$$

$$(M^{0,i(6)})_{\tilde{\nu}_L\phi_1^0} = h_{\chi_L}\mu_1\sigma_R,$$

$$(M^{0,i(6)})_{\tilde{\nu}_L\chi_2^0} = 2h_{\chi_L}h_{\Delta}\sigma_Rv_{\Delta},$$

$$(M^{0,i(6)})_{\tilde{\nu}_L\Delta^0} = -2h_{\chi_L}h_{\Delta}\sigma_R\kappa_2,$$

$$(M^{0,i(6)})_{\tilde{\nu}_R^*\tilde{\nu}_R^*} = 4A_{\Delta}v_{\Delta} + 4h_{\Delta}\mu_2v_{\delta},$$

$$(M^{0,i(6)})_{\tilde{\nu}_R^*\delta^0} = 2A_{\Delta}\sigma_R,$$

$$(M^{0,i(6)})_{\tilde{\nu}_R^*\delta^0} = -2h_{\Delta}\mu_2\sigma_R,$$

$$(M^{0,i(6)})_{\phi_1^0\phi_1^0} = m_{\phi\chi}^2\frac{\kappa_2}{\kappa_1},$$

$$(M^{0,i(6)})_{\phi_1^0\chi_2^0} = m_{\phi\chi}^2,$$

$$(M^{0,i(6)})_{\chi_2^0\chi_2^0} = m_{\phi\chi}^2\frac{\kappa_1}{\kappa_2},$$

$$(M^{0,i(6)})_{\Delta^0\Delta^0} = m_{\Delta\delta}^2\frac{v_{\Delta}}{v_{\Delta}} + \sigma_R^2\frac{A_{\Delta}}{v_{\Delta}},$$

$$(M^{0,i(6)})_{\Delta^0\delta^0} = m_{\Delta\delta}^2,$$

$$(M^{0,i(6)})_{\delta^0\delta^0} = m_{\Delta\delta}^2,$$

$$(M^{0,i(6)})_{\delta^0\delta^0} = m_{\Delta\delta}^2\frac{v_{\Delta}}{v_{\delta}} + h_{\Delta}\mu_2\frac{\sigma_R^2}{v_{\delta}}.$$

$$(A6)$$

The singly charged mass matrix consists of one three by three block and one five by five block as follows:

$$\begin{split} M_{\tilde{e}_L^*,\phi_2^+,\chi_1^+}^2 &= \\ \begin{pmatrix} m_6^2 + h_{\chi L}^2 \sigma_R^2 - \frac{1}{4} (g_L^2 - g_{B-L}^2) \kappa_{dif}^2 + \frac{1}{2} g_{B-L}^2 \rho_{dif}^2 & \mu_1 h_{\chi L} \sigma_R & 2 h_{\chi L} h_{\Delta} \sigma_R v_{\Delta} \\ \mu_1 h_{\chi L} \sigma_R & m_{\phi \chi}^2 \frac{\kappa_2}{\kappa_1} + \frac{1}{2} g_L^2 \kappa_2^2 & m_{\phi \chi}^2 + \frac{1}{2} g_L^2 \kappa_1 \kappa_2 \\ 2 h_{\chi L} h_{\Delta} \sigma_R v_{\Delta} & m_{\phi \chi}^2 + \frac{1}{2} g_L^2 \kappa_1 \kappa_2 & m_{\phi \chi}^2 \frac{\kappa_1}{\kappa_2} + \frac{1}{2} g_L^2 \kappa_1^2 \end{pmatrix} \end{split}$$

$$(A7)$$

and
$$(M^2_{\tilde{e}_{*}^*,\phi_1^+,\chi_2^+,\Delta^+,\delta^+})_{ij} \equiv (M^{+(5)})_{ij}$$
 with

$$(M^{+(5)})_{\tilde{e}_{R}^{*},\phi_{1}^{+},\chi_{2}^{+},\Delta+,\delta+})_{ij} = (M^{+(5)})_{ij} \text{ with }$$

$$(M^{+(5)})_{\tilde{e}_{R}^{*},\tilde{e}_{R}^{-}} = -4h_{\Delta}^{2}v_{\Delta}^{2} + 2h_{\Delta}\mu_{2}v_{\delta} + 2A_{\Delta}v_{\Delta} + g_{R}^{2}\rho_{dif}^{2} + \frac{1}{2}g_{R}^{2}\sigma_{R}^{2} - h_{\chi L}^{2}\kappa_{2}^{2},$$

$$(M^{+(5)})_{\tilde{e}_{R}^{*},\phi_{1}^{-}} = -\frac{1}{2}g_{R}^{2}\sigma_{R}\kappa_{1},$$

$$(M^{+(5)})_{\tilde{e}_{R}^{*},\Delta^{-}} = (h_{\chi L}^{2} - \frac{1}{2}g_{R}^{2})\sigma_{R}\kappa_{2},$$

$$(M^{+(5)})_{\tilde{e}_{R}^{*},\Delta^{-}} = -\sqrt{2}(\frac{1}{2}g_{R}^{2}v_{\Delta} + A_{\Delta} - 2h_{\Delta}^{2}v_{\Delta})\sigma_{R},$$

$$(M^{+(5)})_{\tilde{e}_{R}^{*},\delta^{-}} = \sqrt{2}(\frac{1}{2}g_{R}^{2}v_{\delta} - h_{\Delta}\mu_{2})\sigma_{R},$$

$$(M^{+(5)})_{\phi_{1}^{+},\phi_{1}^{-}} = m_{\phi\chi}^{2}\frac{\kappa_{2}}{\kappa_{1}} + g_{R}^{2}\rho_{dif}^{2} + \frac{1}{2}g_{R}^{2}\kappa_{1}^{2},$$

$$(M^{+(5)})_{\phi_{1}^{+},\chi_{2}^{-}} = m_{\phi\chi}^{2} + \frac{1}{2}g_{R}^{2}\kappa_{1}\kappa_{2},$$

$$(M^{+(5)})_{\phi_{1}^{+},\Delta^{-}} = \frac{1}{\sqrt{2}}g_{R}^{2}\kappa_{1}v_{\Delta},$$

$$(M^{+(5)})_{\phi_{1}^{+},\delta^{-}} = -\frac{1}{\sqrt{2}}g_{R}^{2}\kappa_{1}v_{\delta},$$

$$(M^{+(5)})_{\chi_{2}^{+},\chi_{2}^{-}} = m_{\phi\chi}^{2}\frac{\kappa_{1}}{\kappa_{2}} - g_{R}^{2}\rho_{dif}^{2} + \frac{1}{2}g_{R}^{2}\kappa_{2}^{2} - h_{\chi L}^{2}\sigma_{R}^{2},$$

$$(M^{+(5)})_{\chi_{2}^{+},\Delta^{-}} = \frac{1}{\sqrt{2}}g_{R}^{2}\kappa_{2}v_{\delta},$$

$$(M^{+(5)})_{\lambda^{+},\Delta^{-}} = m_{\Delta\delta}^{2}\frac{v_{\delta}}{v_{\Delta}} - 2h_{\Delta}^{2}\sigma_{R}^{2} + \frac{A_{\Delta}}{v_{\Delta}}\sigma_{R}^{2} + \frac{1}{2}g_{R}^{2}\kappa_{dif}^{2} + g_{R}^{2}(v_{\delta}^{2} + \frac{1}{2}\sigma_{R}^{2}),$$

$$(M^{+(5)})_{\Delta^{+},\Delta^{-}} = m_{\Delta\delta}^{2}\frac{v_{\delta}}{v_{\Delta}} - 2h_{\Delta}^{2}\sigma_{R}^{2} + \frac{A_{\Delta}}{v_{\Delta}}\sigma_{R}^{2} + \frac{1}{2}g_{R}^{2}\kappa_{dif}^{2} + g_{R}^{2}(v_{\delta}^{2} + \frac{1}{2}\sigma_{R}^{2}),$$

$$(M^{+(5)})_{\Delta^{+},\delta^{-}} = m_{\Delta\delta}^{2}\frac{v_{\Delta}}{v_{\delta}} + h_{\Delta}\mu_{2}\frac{\sigma_{R}^{2}}{v_{\delta}} - \frac{1}{2}g_{R}^{2}\kappa_{dif}^{2} + g_{R}^{2}(v_{\Delta}^{2} - \frac{1}{2}\sigma_{R}^{2}).$$

$$(A^{+(5)})_{\delta^{+},\delta^{-}} = m_{\Delta\delta}^{2}\frac{v_{\Delta}}{v_{\delta}} + h_{\Delta}\mu_{2}\frac{\sigma_{R}^{2}}{v_{\delta}} - \frac{1}{2}g_{R}^{2}\kappa_{dif}^{2} + g_{R}^{2}(v_{\Delta}^{2} - \frac{1}{2}\sigma_{R}^{2}).$$

$$(A^{+(5)})_{\delta^{+},\delta^{-}} = m_{\Delta\delta}^{2}\frac{v_{\Delta}}{v_{\delta}} + h_{\Delta}\mu_{2}\frac{\sigma_{R}^{2}}{v_{\delta}} - \frac{1}{2}g_{R}^{2}\kappa_{dif}^{2} + g_{R}^{2}(v_{\Delta}^{2} - \frac{1}{2}\sigma_{R}^{2}).$$

$$(A^{+(5)})_{\delta^{+},\delta^{-}} = m_{\Delta\delta}^{2}\frac{v_{\delta}}{v_{\delta}} + h_{\Delta}\mu_{2}\frac{\sigma_{R}^{2}}{v_{\delta}} - \frac{1}{2}g_{$$

The doubly charged mass matrix can be read from the scalar potential to be

$$\begin{split} M_{\Delta^{++},\delta^{++}}^2 &= \\ \left(\begin{array}{cc} m_{\Delta\delta}^2 \frac{v_{\delta}}{v_{\Delta}} - \sigma_R^2 (4h_{\Delta}^2 - \frac{A_{\Delta}}{v_{\Delta}}) - 2g_R^2 \rho_{dif}^2 & -m_{\Delta\delta}^2 \\ -m_{\Delta\delta}^2 & m_{\Delta\delta}^2 \frac{v_{\Delta}}{v_{\delta}} + h_{\Delta} \mu_2 \frac{\sigma_R^2}{v_{\delta}} + 2g_R^2 \rho_{dif}^2 \end{array} \right). \text{(A9)} \end{split}$$

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particle	mass [GeV]	composition
H_1^0	39	$-0.01\tilde{\nu}_R^r - 0.56\phi_1^{0r} - 0.83\chi_2^{0r} - 0.01\delta^{0r}$
H_2^0	115	$0.57\phi_2^{0r} + 0.82\chi_1^{0r}$
H_3^0	514	$0.41\tilde{\nu}_R^r - 0.01\phi_1^{0r} - 0.01\chi_2^{0r} + 0.64\Delta^{0r} + 0.65\delta^{0r}$
H_4^0	862	$0.90\tilde{\nu}_R^r - 0.16\Delta^{0r} - 0.42\delta^{0r}$
H_5^0	1409	$0.78\tilde{ u}_L^r - 0.52\phi_1^{0r} + 0.35\chi_2^{0r} - 0.02\Delta^{0r} + 0.01\delta^{0r}$
H_6^0	1466	$0.82\phi_2^{0r} - 0.57\chi_1^{0r}$
H_7^0	1515	$0.63\tilde{\nu}_L^r + 0.65\phi_1^{0r} - 0.43\chi_2^{0r} + 0.02\Delta^{0r} - 0.02\delta^{0r}$
H_8^0	2252	$0.16\tilde{\nu}_R^r + 0.02\phi_1^{0r} - 0.03\chi_2^{0r} - 0.75\Delta^{0r} + 0.64\delta^{0r}$
A_1	115	$-0.57\phi_2^{0i} + 0.82\chi_1^{0i}$
A_2	1396	$0.04\tilde{\nu}_L^i + 0.25\tilde{\nu}_R^i - 0.02\phi_1^{0i} - 0.01\chi_2^{0i} - 0.62\Delta^{0i} - 0.74\delta^{0i}$
A_3	1409	$0.77\tilde{\nu}_L^i - 0.01\tilde{\nu}_R^i - 0.53\phi_1^{0i} - 0.35\chi_2^{0i} + 0.03\Delta^{0i} + 0.03\delta^{0i}$
A_4	1466	$0.82\phi_2^{0i} + 0.57\chi_1^{0i}$
A_5	1514	$0.64\tilde{\nu}_L^i + 0.64\phi_1^{0i} + 0.43\chi_2^{0i}$
A_6	2869	$-0.01\tilde{\nu}_L^i + 0.94\tilde{\nu}_R^i + 0.33\Delta^{0i} + 0.04\delta^{0i}$
H_1^{\pm}	121	$-0.02\tilde{e}_R^{\pm} - 0.57\phi_1^{\pm} + 0.82\chi_2^{\pm} - 0.06\Delta^{\pm} - 0.05\delta^{\pm}$
H_2^{\pm}	1329	$0.78\tilde{e}_R^{\pm} - 0.05\phi_1^{\pm} - 0.03\chi_2^{\pm} + 0.20\Delta^{\pm} - 0.58\delta^{\pm}$
H_3^{\pm}	1409	$-0.77\tilde{e}_L^{\pm} + 0.53\phi_2^{\pm} + 0.35\chi_1^{\pm}$
H_4^{\pm}	1467	$+0.07\tilde{e}_R^{\pm} + 0.82\phi_1^{\pm} + 0.57\chi_2^{\pm} - 0.02\Delta^{\pm} - 0.01\delta^{\pm}$
H_5^{\pm}	1516	$0.63\tilde{e}_L^{\pm} + 0.64\phi_2^{\pm} + 0.43\chi_1^{\pm}$
H_6^{\pm}	1928	$-0.53\tilde{e}_R^{\pm} + 0.03\phi_1^{\pm} + 0.03\chi_2^{\pm} + 0.69\Delta^{\pm} - 0.48\delta^{\pm}$
$H_1^{\pm\pm}$	224	$0.68\Delta^{\pm\pm} + 0.73\delta^{\pm\pm}$
$H_2^{\pm\pm}$	1433	$0.73\Delta^{\pm\pm} - 0.68\delta^{\pm\pm}$
$Goldstone_1^0$	0	$-0.11\tilde{\nu}_R^i - 0.48\phi_1^{0i} + 0.71\chi_2^{0i} + 0.36\Delta^{0i} - 0.34\delta^{0i}$
$Goldstone_2^0$	0	$-0.14\tilde{\nu}_R^i + 0.44\phi_1^{0i} - 0.66\chi_2^{0i} + 0.43\Delta^{0i} - 0.41\delta^{0i}$
$Goldstone_1^{\pm}$	0	$-0.55\phi_2^{\pm} + 0.83\chi_1^{\pm}$
$Goldstone_2^{\pm}$	0	$0.31\tilde{e}_R^{\pm} - 0.04\phi_1^{\pm} + 0.07\chi_2^{\pm} + 0.69\Delta^{\pm} + 0.65\delta^{\pm}$

Table 1: Masses and compositions of the scalars and the Goldstone bosons. Parameters are chosen as follows $m_{\phi\chi}=m_{\Delta\delta}=1$ TeV, $\mu_2=100$ GeV, $h_{\Delta}=0.6$, $A_{\Delta}=1700$ GeV, $\tan\beta=1.5$, $v_{\delta}/\sigma_R=1.5$, $v_{\Delta}/v_{\delta}=1.05$, and $h_{\chi L}=0.1$.

particle	mass [GeV]	composition
H_1^0	101	$0.02\tilde{\nu}_R^r + 0.02\phi_1^{0r} + 1.00\chi_2^{0r} + 0.03\delta^{0r}$
H_2^0	181	$0.02\phi_2^{0r} + 1.00\chi_1^{0r}$
H_3^0	514	$-0.01\tilde{\nu}_L^r - 0.41\tilde{\nu}_R^r + 0.03\chi_2^{0r} - 0.64\Delta^{0r} - 0.65\delta^{0r}$
H_4^0	862	$-0.90\tilde{\nu}_R^r + 0.16\Delta^{0r} + 0.42\delta^{0r}$
H_5^0	1317	$1.00\tilde{\nu}_L^r - 0.09\phi_1^{0r} - 0.01\delta^{0r}$
H_{6}^{0}	2252	$-0.16\tilde{\nu}_R^r + 0.02\chi_2^{0r} + 0.75\Delta^{0r} - 0.64\delta^{0r}$
H_{7}^{0}	7070	$1.00\phi_2^{0r} - 0.02\chi_1^{0r}$
H_8^0	7099	$-0.09\tilde{\nu}_L^r - 1.00\phi_1^{0r} + 0.02\chi_2^{0r}$
A_1	182	$-0.02\phi_2^{0i} + 1.00\chi_1^{0i}$
A_2	1317	$1.00\tilde{\nu}_L^i + 0.01\tilde{\nu}_R^i - 0.09\phi_1^{0i}$
A_3	1397	$-0.01\tilde{\nu}_L^i + 0.25\tilde{\nu}_R^i - 0.62\Delta^{0i} - 0.74\delta^{0i}$
A_4	2869	$0.01\tilde{\nu}_L^i - 0.94\tilde{\nu}_R^i - 0.33\Delta^{0i} - 0.04\delta^{0i}$
A_5	7070	$1.00\phi_2^{0i} + 0.02\chi_1^{0i}$
A_6	7099	$0.09\tilde{\nu}_L^i + 1.00\phi_1^{0i} + 0.02\chi_2^{0i}$
H_1^{\pm}	206	$-0.02\tilde{e}_R^{\pm} - 0.02\phi_1^{\pm} + 1.00\chi_2^{\pm} - 0.07\Delta^{\pm} - 0.03\delta^{\pm}$
H_2^{\pm}	1320	$-1.00\tilde{e}_L^{\pm} + 0.09\phi_2^{\pm}$
H_3^{\pm}	1330	$0.79\tilde{e}_R^{\pm} + 0.01\chi_2^{\pm} + 0.20\Delta^{\pm} - 0.58\delta^{\pm}$
H_4^{\pm}	1927	$0.53\tilde{e}_R^{\pm} - 0.02\chi_2^{\pm} - 0.69\Delta^{\pm} + 0.48\delta^{\pm}$
H_5^{\pm}	7070	$1.00\phi_1^{\pm} + 0.02\chi_2^{\pm}$
H_6^{\pm}	7099	$0.09\tilde{e}_L^{\pm} + 1.00\phi_2^{\pm} + 0.02\chi_1^{\pm}$
$H_1^{\pm\pm}$	225	$0.69\Delta^{\pm\pm} + 0.73\delta^{\pm\pm}$
$H_2^{\pm\pm}$	1433	$0.73\Delta^{\pm\pm} - 0.69\delta^{\pm\pm}$
$Goldstone_1^0$	0	$-0.02\tilde{\nu}_R^i - 0.02\phi_1^{0i} + 1.00\chi_2^{0i} + 0.07\Delta^{0i} - 0.06\delta^{0i}$
$Goldstone_2^0$	0	$-0.22\tilde{\nu}_R^i + 0.01\chi_2^{0i} + 0.71\Delta^{0i} - 0.67\delta^{0i}$
$Goldstone_1^{\pm}$	0	$-0.02\phi_2^{\pm} + 1.00\chi_1^{\pm}$
$Goldstone_2^{\pm}$	0	$0.31\tilde{e}_R^{\pm} + 0.08\chi_2^{\pm} + 0.69\Delta^{\pm} + 0.65\delta^{\pm}$

Table 2: Same as Table 1, except for $\tan\beta=50.$

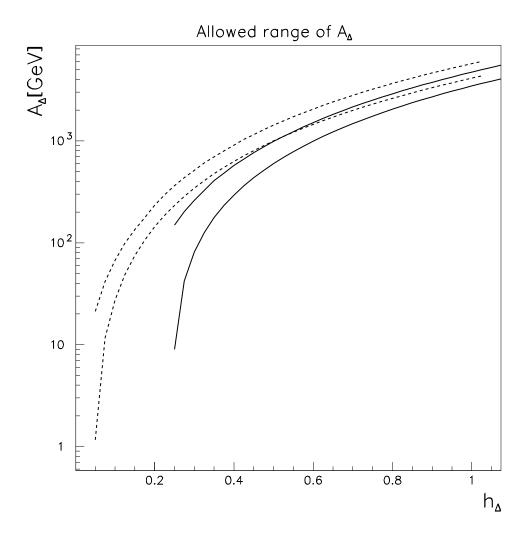


Figure 1: The upper and lower limits of A_{Δ} as a function of h_{Δ} . The parameters are chosen as follows: $m_{\phi\chi} = m_{\Delta\delta} = 1$ TeV, $\tan\beta = 50$, $v_{\delta}/\sigma_R = 1.5$, $v_{\Delta}/v_{\delta} = 1.05$, and $h_{\chi L} = 0.1$. The solid curves correspond to $\mu_2 = 1$ TeV and the dashed curves to $\mu_2 = 100$ GeV.

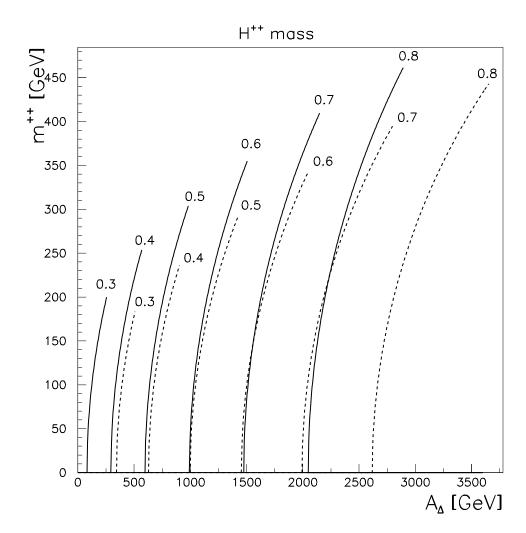


Figure 2: The mass of the light doubly charged scalar as a function of the allowed A_{Δ} values for $h_{\Delta}=0.3,\ldots,0.8$. The parameters are chosen as follows: $m_{\phi\chi}=m_{\Delta\delta}=1$ TeV, $\tan\beta=50,\ v_{\delta}/\sigma_R=1.5,\ v_{\Delta}/v_{\delta}=1.05,\ {\rm and}\ h_{\chi L}=0.1$. The solid curves correspond to $\mu_2=1$ TeV and the dashed curves to $\mu_2=100$ GeV.